SOLUTION OF EXERCISE # 3.3

Exercise # 3.3

Prove the following identities.

Q.1: $1-2\sin^2\theta = 2\cos^2\theta - 1$

(IIA-2017), (IA-2018), (IIA-2021)

Sol. L.H.S. $= 1-2\sin^2\theta$

$= 1-2(1-\cos^2\theta) \quad \because \{\sin^2\theta = 1-\cos^2\theta\}$

$= 1-2+2\cos^2\theta$

$= 2\cos^2\theta - 1 =$ R.H.S. **Proved**

Q.2: $\cos^4\theta - \sin^4\theta = 1-2\sin^2\theta$

(IA-2016), (IA-2017), (IIA-2020), (IA-2021)

Sol. L.H.S. $= \cos^4\theta - \sin^4\theta = (\cos^2\theta)^2 - (\sin^2\theta)^2$

$= (\cos^2\theta - \sin^2\theta)(\cos^2\theta + \sin^2\theta)$

$= (1-\sin^2\theta - \sin^2\theta)(1) \quad \because \{\cos^2\theta = 1-\sin^2\theta\}$

$= 1-2\sin^2\theta =$ R.H.S. **Proved**

Q.3: $\dfrac{1}{\operatorname{cosec}^2\theta} + \dfrac{1}{\sec^2\theta} = 1$

Sol. L.H.S. $= \dfrac{1}{\operatorname{cosec}^2\theta} + \dfrac{1}{\sec^2\theta}$

$= \sin^2\theta + \cos^2\theta = 1 =$ R.H.S. **Proved**

Q.4: $\dfrac{1}{\tan\theta + \cot\theta} = \sin\theta.\cos\theta$ (IA-2021)

Sol. L.H.S. $= \dfrac{1}{\tan\theta+\cot\theta} = \dfrac{1}{\dfrac{\sin\theta}{\cos\theta}+\dfrac{\cos\theta}{\sin\theta}} = \dfrac{1}{\dfrac{\sin^2\theta+\cos^2\theta}{\cos\theta\sin\theta}}$

$= \dfrac{\sin\theta\cos\theta}{\sin^2\theta+\cos^2\theta} = \dfrac{\sin\theta\cos\theta}{1} \quad \because \{\sin^2\theta+\cos^2\theta = 1\}$

$= \sin\theta\cos\theta =$ R.H.S. **Proved**

SOLUTION OF EXERCISE # 3.3

Q.5:

$$(\sec\theta - \tan\theta)^2 = \frac{1-\sin\theta}{1+\sin\theta}$$

Sol. L.H.S.

$$= (\sec\theta - \tan\theta)^2$$

$$= \left(\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right)^2$$

$$= \left(\frac{1-\sin\theta}{\cos\theta}\right)^2$$

$$= \frac{(1-\sin\theta)^2}{\cos^2\theta}$$

$$= \frac{(1-\sin\theta)^2}{1-\sin^2\theta}$$

$$= \frac{(1-\sin\theta)^2}{(1)^2-(\sin\theta)^2}$$

$$= \frac{(1-\sin\theta)^2}{(1-\sin\theta)(1+\sin\theta)}$$

$$= \frac{1-\sin\theta}{1+\sin\theta} = \text{R.H.S.}$$

Proved.

Q.6:

$$(\operatorname{cosec}\theta - \cot\theta)^2 = \frac{1-\cos\theta}{1+\cos\theta}$$

(IA-2017), (IIA-2019)

Sol. L.H.S.

$$= (\operatorname{cosec}\theta - \cot\theta)^2$$

$$= \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2$$

$$= \left(\frac{1-\cos\theta}{\sin\theta}\right)^2$$

$$= \frac{(1-\cos\theta)^2}{\sin^2\theta}$$

$$= \frac{(1-\cos\theta)^2}{1-\cos^2\theta}$$

$$= \frac{(1-\cos\theta)^2}{(1)^2-(\cos\theta)^2}$$

$$= \frac{(1-\cos\theta)^2}{(1-\cos\theta)(1+\cos\theta)}$$

$$= \frac{1-\cos\theta}{1+\cos\theta} = \text{R.H.S.}$$

Proved.

Q.7: $(1-\sin^2\theta)(1+\tan^2\theta) = 1$

Sol. L.H.S. $= (1-\sin^2\theta)(1+\tan^2\theta)$

$$= \cos^2\theta \sec^2\theta \qquad \left\{\begin{array}{ll} \because & 1+\tan^2\theta = \sec^2\theta \\ \& & 1-\sin^2\theta = \cos^2\theta \end{array}\right\}$$

$$= \cos^2\theta \frac{1}{\cos^2\theta} = 1 = \text{R.H.S.}$$

Proved

SOLUTION OF EXERCISE # 3.3

Q.8: $\dfrac{1}{1+\sin\theta}+\dfrac{1}{1-\sin\theta}=2\sec^2\theta$

(IIA-2016), (IIA-2018), (IA-2021), (IIA-2021)

Sol. $\text{L.H.S.}=\dfrac{1}{1+\sin\theta}+\dfrac{1}{1-\sin\theta}=\dfrac{1(1-\sin\theta)+1(1+\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}$

$=\dfrac{1-\sin\theta+1+\sin\theta}{(1)^2-(\sin\theta)^2}=\dfrac{2}{1-\sin^2\theta}$

$=\dfrac{2}{\cos^2\theta}$ $\quad\because 1-\sin^2\theta=\cos^2\theta$

$=2\sec^2\theta=\text{R.H.S.}$ **Proved.**

Q.9: $\sqrt{\dfrac{1-\sin\theta}{1+\sin\theta}}=\sec\theta-\tan\theta$

(IA-2019), (IIA-2020), (IA-2022)

Sol. $\text{L.H.S.}=\sqrt{\dfrac{1-\sin\theta}{1+\sin\theta}}$

$=\sqrt{\dfrac{1-\sin\theta}{1+\sin\theta}\times\dfrac{1-\sin\theta}{1-\sin\theta}}$

$=\sqrt{\dfrac{(1-\sin\theta)^2}{(1)^2-(\sin\theta)^2}}$

$=\sqrt{\dfrac{(1-\sin\theta)^2}{1-\sin^2\theta}}$

$=\sqrt{\dfrac{(1-\sin\theta)^2}{\cos^2\theta}}$

$=\dfrac{1-\sin\theta}{\cos\theta}$

$=\dfrac{1}{\cos\theta}-\dfrac{\sin\theta}{\cos\theta}$

$=\sec\theta-\tan\theta=\text{R.H.S}$

Proved.

Q.10: $\sqrt{\dfrac{1+\cos\theta}{1-\cos\theta}}=\operatorname{cosec}\theta+\cot\theta$

Sol. $\text{L.H.S.}=\sqrt{\dfrac{1+\cos\theta}{1-\cos\theta}}$

$=\sqrt{\dfrac{1+\cos\theta}{1-\cos\theta}\times\dfrac{1+\cos\theta}{1+\cos\theta}}$

$=\sqrt{\dfrac{(1+\cos\theta)^2}{(1)^2-(\cos\theta)^2}}$

$=\sqrt{\dfrac{(1+\cos\theta)^2}{1-\cos^2\theta}}$

$=\sqrt{\dfrac{(1+\cos\theta)^2}{\sin^2\theta}}$

$=\dfrac{1+\cos\theta}{\sin\theta}=\dfrac{1}{\sin\theta}+\dfrac{\cos\theta}{\sin\theta}$

$=\operatorname{cosec}\theta+\cot\theta=\text{R.H.S.}$

Proved.

SOLUTION OF EXERCISE # 3.3

Q.11: $\dfrac{\cot^2\theta-1}{\cot^2\theta+1}=2\cos^2\theta-1$ (IIA-2017)

Sol. L.H.S. $=\dfrac{\cot^2\theta-1}{\cot^2\theta+1}$

$$=\frac{\dfrac{\cos^2\theta}{\sin^2\theta}-1}{\operatorname{cosec}^2\theta} \qquad \because\ 1+\cot^2\theta=\operatorname{cosec}^2\theta$$

$$=\left(\frac{\cos^2\theta-\sin^2\theta}{\sin^2\theta}\right).\sin^2\theta$$

$$=\cos^2\theta-\sin^2\theta$$

$$=\cos^2\theta-(1-\cos^2\theta) \qquad \because\ \sin^2\theta=1-\cos^2\theta$$

$$=\cos^2\theta-1+\cos^2\theta=2\cos^2\theta-1=\text{R.H.S.}$$ **Proved**

Q.12: $\dfrac{\tan\theta}{1-\cot\theta}+\dfrac{\cot\theta}{1-\tan\theta}=\sec\theta.\operatorname{cosec}\theta+1$

Sol. L.H.S. $=\dfrac{\tan\theta}{1-\cot\theta}+\dfrac{\cot\theta}{1-\tan\theta}$ (IA-2019), (IA-2022)

$$=\frac{\dfrac{\sin\theta}{\cos\theta}}{1-\dfrac{\cos\theta}{\sin\theta}}+\frac{\dfrac{\cos\theta}{\sin\theta}}{1-\dfrac{\sin\theta}{\cos\theta}} \qquad \because\left\{\begin{aligned}\tan\theta&=\frac{\sin\theta}{\cos\theta}\\ \cot\theta&=\frac{\cos\theta}{\sin\theta}\end{aligned}\right\}$$

$$=\frac{\dfrac{\sin\theta}{\cos\theta}}{\dfrac{\sin\theta-\cos\theta}{\sin\theta}}+\frac{\dfrac{\cos\theta}{\sin\theta}}{\dfrac{\cos\theta-\sin\theta}{\cos\theta}}$$

$$=\left(\frac{\sin\theta}{\cos\theta}\right)\left(\frac{\sin\theta}{\sin\theta-\cos\theta}\right)+\left(\frac{\cos\theta}{\sin\theta}\right)\left(\frac{\cos\theta}{\cos\theta-\sin\theta}\right)$$

$$=\frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)}+\frac{\cos^2\theta}{\sin\theta(\cos\theta-\sin\theta)}$$

SOLUTION OF EXERCISE # 3.3

$$=\frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)}+\frac{\cos^2\theta}{-\sin\theta[-(\cos\theta-\sin\theta)]}$$

$$=\frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)}-\frac{\cos^2\theta}{\sin\theta(\sin\theta-\cos\theta)}$$

$$=\frac{\sin^3\theta-\cos^3\theta}{\cos\theta\sin\theta(\sin\theta-\cos\theta)}$$

$$=\frac{\cancel{(\sin\theta-\cos\theta)}(\sin^2\theta+\sin\theta\cos\theta+\cos^2\theta)}{\cos\theta\sin\theta\cancel{(\sin\theta-\cos\theta)}}$$

$$=\frac{1+\sin\theta\cos\theta}{\cos\theta\sin\theta} \qquad \because\{\sin^2\theta+\cos^2\theta=1\}$$

$$=\frac{1}{\cos\theta\sin\theta}+\frac{\sin\theta\cos\theta}{\cos\theta\sin\theta}$$

$$=\sec\theta\ \operatorname{cosec}\theta+1=\text{R.H.S.}$$

Proved.

Q.13: $\dfrac{\sec\theta-\tan\theta}{\sec\theta+\tan\theta}=1-2\sec\theta\tan\theta+2\tan^2\theta$

(IIA-2016)

Sol. L.H.S.$=\dfrac{\sec\theta-\tan\theta}{\sec\theta+\tan\theta}$

$$=\frac{\sec\theta-\tan\theta}{\sec\theta+\tan\theta}\times\frac{\sec\theta-\tan\theta}{\sec\theta-\tan\theta}$$

$$=\frac{(\sec\theta-\tan\theta)^2}{(\sec\theta)^2-(\tan\theta)^2}$$

$$=\frac{(\sec\theta)^2-2(\sec\theta)(\tan\theta)+(\tan\theta)^2}{\sec^2\theta-\tan^2\theta}$$

$$=\frac{1+\tan^2\theta-2\sec\theta\tan\theta+\tan^2\theta}{1+\cancel{\tan^2\theta}-\cancel{\tan^2\theta}} \quad \because\{\sec^2\theta=1+\tan^2\theta\}$$

$$=1-2\sec\theta\tan\theta+2\tan^2\theta=\text{R.H.S.}$$

Proved.

Q.14: $\dfrac{1+\tan^2\theta}{1+\cot^2\theta}=\dfrac{(1-\tan\theta)^2}{(1-\cot\theta)^2}$

(IIA-2018)

SOLUTION OF EXERCISE # 3.3

Sol. $$\text{R.H.S.} = \frac{(1-\tan\theta)^2}{(1-\cot\theta)^2} = \frac{\left(1-\frac{\sin\theta}{\cos\theta}\right)^2}{\left(1-\frac{\cos\theta}{\sin\theta}\right)^2} = \frac{\left(\frac{\cos\theta-\sin\theta}{\cos\theta}\right)^2}{\left(\frac{\sin\theta-\cos\theta}{\sin\theta}\right)^2}$$

$$= \frac{(-(\sin\theta-\cos\theta))^2}{\cos^2\theta}\cdot\frac{\sin^2\theta}{(\sin\theta-\cos\theta)^2}$$

$$= \frac{\cancel{(\sin\theta-\cos\theta)}\sin^2\theta}{\cos^2\theta\cancel{(\sin\theta-\cos\theta)}} = \frac{\sin^2\theta}{\cos^2\theta}$$

$$= \frac{\sec^2\theta}{\operatorname{cosec}^2\theta} = \frac{1+\tan^2\theta}{1+\cot^2\theta} = \text{L.H.S.}$$ **Proved.**

Q.15: $(1-\tan\theta)^2+(1-\cot\theta)^2=(\sec\theta-\operatorname{cosec}\theta)^2$

Sol. $$\text{L.H.S.} = (1-\tan\theta)^2+(1-\cot\theta)^2$$

$$=(1)^2+(\tan\theta)^2-2(1)(\tan\theta)+(1)^2+(\cot\theta)^2-2(1)(\cot\theta)$$

$$=1+\tan^2\theta-2\tan\theta+1+\cot^2\theta-2\cot\theta$$

$$=\sec^2\theta-2\tan\theta+\operatorname{cosec}^2\theta-2\cot\theta$$

$$=\sec^2\theta+\operatorname{cosec}^2\theta-2\tan\theta-2\cot\theta$$

$$=\sec^2\theta+\operatorname{cosec}^2\theta-2(\tan\theta+\cot\theta)$$

$$=\sec^2\theta+\operatorname{cosec}^2\theta-2\left(\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}\right)$$

$$=\sec^2\theta+\operatorname{cosec}^2\theta-2\left(\frac{\sin^2\theta+\cos^2\theta}{\cos\theta\sin\theta}\right)$$

$$=\sec^2\theta+\operatorname{cosec}^2\theta-2\left(\frac{1}{\cos\theta\sin\theta}\right) \quad \because\{\sin^2\theta+\cos^2\theta=1\}$$

$$=\sec^2\theta+\operatorname{cosec}^2\theta-2\sec\theta\operatorname{cosec}\theta$$

$$=(\sec\theta-\operatorname{cosec}\theta)^2=\text{R.H.S.}$$ **Proved.**

SOLUTION OF EXERCISE # 3.3

Q.16: $\dfrac{\cos^3 t-\sin^3 t}{\cos t-\sin t}=1+\sin t\ \cos t$

Sol. $\text{L.H.S.}=\dfrac{\cos^3 t-\sin^3 t}{\cos t-\sin t}$

$$=\frac{(\cos t-\sin t)(\cos^2 t+\cos t\ \sin t+\sin^2 t)}{(\cos t-\sin t)}$$

$$=\cos^2 t+\sin^2 t+\cos t\ \sin t$$

$$=1+\sin t\ \cos t=\text{R.H.S.}$$

Proved.

Q.17: $\sec^2 A+\tan^2 A=\left(1-\sin^4 A\right)\sec^4 A$

Sol. $\text{L.H.S.}=\sec^2 A+\tan^2 A$

$$=\frac{1}{\cos^2 A}+\frac{\sin^2 A}{\cos^2 A}$$

$$=\frac{1+\sin^2 A}{\cos^2 A}$$

$$=\frac{1+\sin^2 A}{\cos^2 A}\times\frac{1-\sin^2 A}{1-\sin^2 A}$$

$$=\frac{(1)^2-(\sin^2 A)^2}{\cos^2 A\ .\ \cos^2 A}$$

$$=\frac{\left(1-\sin^4 A\right)}{\cos^4 A}=\left(1-\sin^4 A\right)\sec^4 A=\text{R.H.S.}$$

Proved.

Q.18: $\dfrac{\sec x-\cos x}{1+\cos x}=\sec x-1$

Sol. $\text{L.H.S.}=\dfrac{\sec x-\cos x}{1+\cos x}$

$$=\frac{\frac{1}{\cos x}-\cos x}{1+\cos x}=\frac{\frac{1-\cos^2 x}{\cos x}}{1+\cos x}=\frac{(1)^2-(\cos x)^2}{\cos x(1+\cos x)}$$

SOLUTION OF EXERCISE # 3.3

$$=\frac{(1-\cos x)(1+\cos x)}{\cos x(1+\cos x)}=\frac{1-\cos x}{\cos x}$$

$$=\frac{1}{\cos x}-\frac{\cos x}{\cos x}=\sec x-1=\text{R.H.S.}$$ **Proved.**

Q.19: $\dfrac{\sin x+\cos x}{\tan^2 x-1}=\dfrac{\cos^2 x}{\sin x-\cos x}$

Sol. $\text{L.H.S.}=\dfrac{\sin x+\cos x}{\tan^2 x-1}$

$$=\frac{\sin x+\cos x}{\frac{\sin^2 x}{\cos^2 x}-1}=\frac{\sin x+\cos x}{\frac{\sin^2 x-\cos^2 x}{\cos^2 x}}$$

$$=\frac{(\sin x+\cos x).\cos^2 x}{\sin^2 x-\cos^2 x}=\frac{(\sin x+\cos x).\cos^2 x}{(\sin x-\cos x)(\sin x+\cos x)}$$

$$=\frac{\cos^2 x}{\sin x-\cos x}=\text{R.H.S.}$$ **Proved.**

Q.20: $(1+\sin\theta)(1-\sin\theta)=\dfrac{1}{\sec^2\theta}$ (IIA-2019)

Sol. $\text{L.H.S.}=(1+\sin\theta)(1-\sin\theta)$

$$=(1)^2-(\sin\theta)^2=1-\sin^2\theta$$

$$=\cos^2\theta=\frac{1}{\sec^2\theta}=\text{R.H.S.}$$ **Proved.**

Q.21: $\dfrac{\tan\theta}{\sec\theta-1}+\dfrac{\tan\theta}{\sec\theta+1}=2\operatorname{cosec}\theta$

Sol. $\text{L.H.S.}=\dfrac{\tan\theta}{\sec\theta-1}+\dfrac{\tan\theta}{\sec\theta+1}$

$$=\tan\theta\left(\frac{1}{\sec\theta-1}+\frac{1}{\sec\theta+1}\right)$$

SOLUTION OF EXERCISE # 3.3

$$= \tan\theta\left(\frac{1(\sec\theta+1)+1(\sec\theta-1)}{(\sec\theta-1)(\sec\theta+1)}\right) \quad \left\{\text{By taking L.C.M}\right\}$$

$$= \tan\theta\left(\frac{\sec\theta+1+\sec\theta-1}{(\sec\theta)^2-(1)^2}\right) = \tan\theta\left(\frac{2\sec\theta}{\sec^2\theta-1}\right)$$

$$= \tan\theta\left(\frac{2\sec\theta}{1+\tan^2\theta-1}\right) \quad \because \sec^2\theta = 1+\tan^2\theta$$

$$= \tan\theta\left(\frac{2\sec\theta}{\tan^2\theta}\right) = \frac{2\sec\theta}{\tan\theta} = 2\sec\theta.\cot\theta$$

$$= 2\frac{1}{\cos\theta}\times\frac{\cos\theta}{\sin\theta} = 2\frac{1}{\sin\theta} = 2\operatorname{cosec}\theta = \text{R.H.S.}$$ **Proved.**

Q.22: If $m = \tan\theta + \sin\theta$ and $n = \tan\theta - \sin\theta$, then prove that $m^2 - n^2 = 4\sqrt{mn}$. (IA-2016), (IIA-2021)

Sol. L.H.S.$= m^2 - n^2 = (\tan\theta + \sin\theta)^2 - (\tan\theta - \sin\theta)^2$

$$= (\tan^2\theta + \sin^2\theta + 2\tan\theta\sin\theta) - (\tan^2\theta + \sin^2\theta - 2\tan\theta\sin\theta)$$

$$= \tan^2\theta + \sin^2\theta + 2\tan\theta\sin\theta - \tan^2\theta - \sin^2\theta + 2\tan\theta\sin\theta$$

$$= 4\tan\theta\sin\theta \rightarrow (i)$$

$$\text{R.H.S.} = 4\sqrt{mn} = 4\sqrt{(\tan\theta+\sin\theta)(\tan\theta-\sin\theta)}$$

$$= 4\sqrt{\tan^2\theta-\sin^2\theta}$$

$$= 4\sqrt{\frac{\sin^2\theta}{\cos^2\theta}-\sin^2\theta} = 4\sqrt{\frac{\sin^2\theta-\sin^2\theta\cos^2\theta}{\cos^2\theta}}$$

$$= 4\sqrt{\frac{\sin^2\theta(1-\cos^2\theta)}{\cos^2\theta}} = 4\sqrt{\frac{\sin^2\theta.\sin^2\theta}{\cos^2\theta}}$$

$$= 4\sqrt{\tan^2\theta\sin^2\theta} = 4\tan\theta\sin\theta \rightarrow (ii)$$

From equation eq.(i) & eq.(ii), we have

L.H.S. = R.H.S. **Proved.**